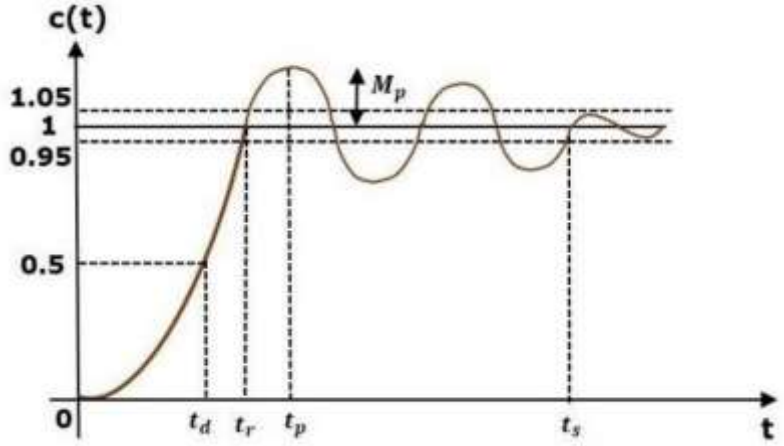
EXPERIMENT No. 7

Time Response Analysis of First and Second Order Systems using MATLAB

AIM: To obtain time response of a second order system in case of under damped systems, for different damping factors.

THEORY: The time response of control system consists of two parts. Transient response and steady state response. Most of the control systems use time as its independent variable. All the time domain specifications are represented in the figure. The response up to the settling time is known as transient response and the response after the settling time is known as steady state response.



Analysis of response means to see the variation of output with respect to time. The output of the system takes some finite time to reach to its final value. Every system has a tendency to oppose the oscillatory behaviour of the system which is called damping. The damping is measured by a factor called damping ratio (ξ) of the system.

- If the damping is very high then there will not be any oscillations in the output. The output is purely exponential. Such system is called an over damped system ($1 < \xi < \infty$)
- If the damping is less compared to over damped case, the system is called a critically damped system ($\xi = 1$)
- If the damping is very less, the system is called under damped system($0 < \xi < 1$)
- With no damping system is undamped. ($\xi = 0$)

Time domain specifications:

- Damped Natural Frequency: $w_d = w_n\sqrt{1-\xi^2}$
- Rise Time: It is the time required for the response to rise from 0 to 100% of the final value for the underdamped system $t_r = \frac{\pi-\theta}{w_n\sqrt{1-\xi^2}}$; $\theta = cos^{-1}(\xi)$
- Peak time: It is the time required for the response to reach the peak of time response or the peak overshoot $t_p = \frac{\pi}{w_n\sqrt{1-\xi^2}}$
- Settling Time for 2% tolerance band: It is the time required for the response to reach and stay within a specified tolerance band (2% or 5%) of its final value $t_s = \frac{4}{\xi w_n}$
- Percent Overshoot: $M_p = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}$

% Second-Order Underdamped System Example

```
   % Given System
wn = 10; % Natural Frequency
zeta = [0.1  0.5 0.9]; % Damping Ratio

   % Calculate Time Response Specification for Different Damping Factors
for i = 1:length(zeta)
theta(i)= acos(zeta(i));
wd = wn*sqrt(1-zeta(i)^2);    % Calculate Damped Natural Frequency

    % Calculate Time Response Specifications
tr = (pi-theta(i))/wd;
tp= pi/wd;
ts = 4/(zeta(i)*wn);
Mp = exp(-zeta(i)*pi/sqrt(1-zeta(i)^2));

    % Display Results
fprintf('Damping Ratio = %.1f\n', zeta(i));
fprintf('Damped Natural Frequency = %.2f rad/s\n', wd);
fprintf('Rise Time = %.2f s\n', tr);
fprintf('Peak Time = %.2f s\n', tp);
fprintf('Settling Time = %.2f s\n', ts);
fprintf('Percent Overshoot = %.2f %%\n\n', Mp*100);
end
```

RESULT:

```
Damping Ratio = 0.1
Damped Natural Frequency = 9.95 rad/s
Rise Time = 0.17 s
Peak Time = 0.32 s
Settling Time = 4.00 s
Percent Overshoot = 72.92 %

Damping Ratio = 0.5
Damped Natural Frequency = 8.66 rad/s
Rise Time = 0.24 s
Peak Time = 0.36 s
Settling Time = 0.80 s
Percent Overshoot = 16.30 %

Damping Ratio = 0.9
Damped Natural Frequency = 4.36 rad/s
Rise Time = 0.62 s
Peak Time = 0.72 s
Settling Time = 0.44 s
Percent Overshoot = 0.15 %
```